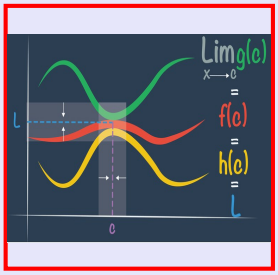
Math 261

Spring 2023

Lecture 5



Feb 19-8:47 AM

Class QZ 1

Solve $3x^2 - 5x = 8$ using the quadratic formula.

$ax^2 + bx + c = 0$

$3x^2 - 5x - 8 = 0$

$a = 3$

$b = -5$

$c = -8$

$b^2 - 4ac = (-5)^2 - 4(3)(-8) = 25 + 86 = 121$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{121}}{2(3)} = \frac{5 \pm 11}{6}$$

$$x = \frac{5 + 11}{6} = \frac{16}{6} = \frac{8}{3}$$

$$x = \frac{5 - 11}{6} = \frac{-6}{6} = -1$$

$\left\{ -1, \frac{8}{3} \right\}$

Solution Set

Feb 9-9:39 AM

Introduction to limits:

$\lim_{x \to a^+} f(x) = L_1$

x approaches a from the right

$\lim_{x \to a^-} f(x) = L_2$

x approaches a from the left

If $L_1 = L_2$, then

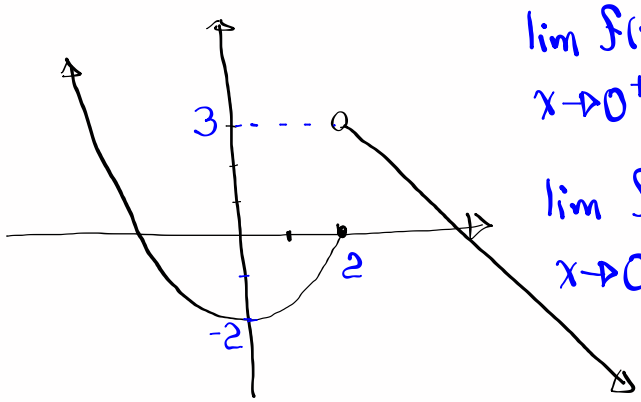
$\lim_{x \to a} f(x) = L$ where $L = L_1 = L_2$

If $L_1 \neq L_2$, then

$\lim_{x \to a} f(x)$ D.N.E.

Feb 13-8:52 AM

Consider the graph of f(x) below



$\lim_{x \to 0^+} f(x) = -2$

$\lim_{x \to 0^-} f(x) = -2$

$\lim_{x \to 0} f(x) = -2$

$\lim_{x \to 2^+} f(x) = 3$

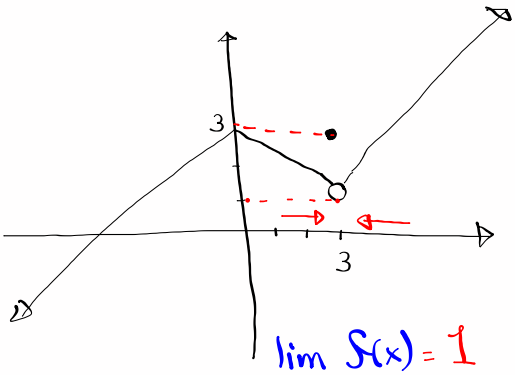
$\lim_{x \to 2^-} f(x) = 0$

Since $3 \neq 0$

$\lim_{x \to 2} f(x)$ D.N.E.

Feb 13-8:55 AM

Consider the graph of $f(x)$ below:



$\lim_{x \to 3^+} f(x) = 1$ L_1

$\lim_{x \to 3^-} f(x) = 1$ L_2

$\lim_{x \to 3} f(x) = 1$

$f(x)$ is <u>not</u> cont. at $x=3$.

$f(3) = 3$

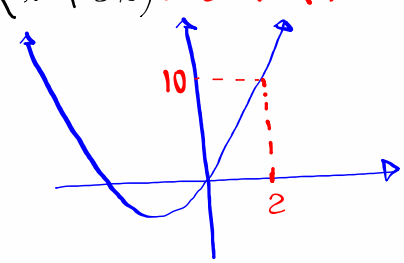
If $\lim_{x \to a} f(x) = f(a)$, then $f(x)$ is continuous at $x=a$.

Feb 13-9:00 AM

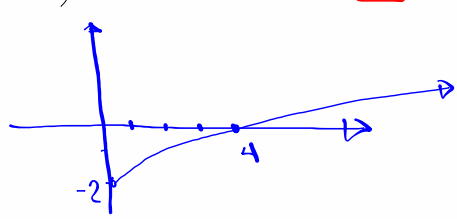
How to evaluate limits:

Simply plug it in, and hope for the best.

$\lim_{x \to 2} (x^2 + 3x) = 2^2 + 3(2) = 4 + 6 = \boxed{10}$



Evaluate $\lim_{x \to 4} (\sqrt{x} - 2) = \sqrt{4} - 2 = 2 - 2 = \boxed{0}$



Feb 13-9:06 AM

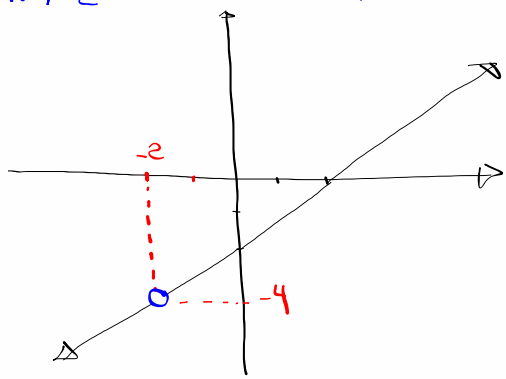
Evaluate $\lim_{x \to -2} \frac{x^2-4}{x+2} = \frac{(-2)^2-4}{-2+2} = \frac{4-4}{-2+2} = \frac{0}{0}$

Indeterminate form

$\to x \neq -2$

Factor & Simplify

$\lim_{x \to -2} \frac{\cancel{(x+2)}(x-2)}{\cancel{x+2}} = \lim_{x \to -2} (x-2) = -2-2 = \boxed{-4}$



Feb 13-9:13 AM

Evaluate $\lim_{x \to 2} \frac{x^3-8}{x^2-4} = \frac{2^3-8}{2^2-4} = \frac{8-8}{4-4} = \frac{0}{0}$ I.F.

Factor & Simplify

$(A-B)(A^2+AB+B^2)$

$\lim_{x \to 2} \frac{x^3-8}{x^2-4} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{(x-2)}(x+2)}$

$x^3-8 = x^3-2^3$

Use A^3-B^3

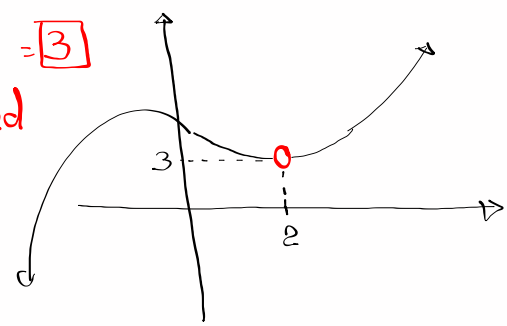
Review

$= \lim_{x \to 2} \frac{x^2+2x+4}{x+2} = \frac{2^2+2(2)+4}{2+2} = \frac{12}{4}$

$= \boxed{3}$

$f(2)$ is undefined

$\lim_{x \to 2} f(x) = 3$



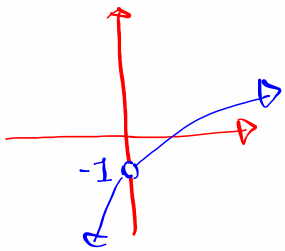
Feb 13-9:19 AM

Evaluate $\lim_{x \to 0} \dfrac{\frac{1}{x+1} - 1}{x} = \dfrac{\frac{1}{0+1} - 1}{0}$

Use LCD $= x+1$ to Simplify $= \dfrac{\frac{1}{1} - 1}{0} = \dfrac{1-1}{0} = \dfrac{0}{0}$ I.F.

$$\lim_{x \to 0} \frac{\frac{1}{x+1} - 1}{x} = \lim_{x \to 0} \frac{\cancel{(x+1)} \cdot \frac{1}{\cancel{x+1}} - (x+1) \cdot 1}{x(x+1)}$$

$$= \lim_{x \to 0} \frac{1 - (x+1)}{x(x+1)} = \lim_{x \to 0} \frac{\cancel{1} - x - \cancel{1}}{x(x+1)}$$



$$= \lim_{x \to 0} \frac{-\cancel{x}}{\cancel{x}(x+1)} = \lim_{x \to 0} \frac{-1}{x+1}$$

$$= \frac{-1}{0+1} = \frac{-1}{1} = \boxed{-1}$$

Feb 13-9:26 AM

Evaluate $\lim_{x \to 9} \dfrac{x-9}{\sqrt{x} - 3} = \dfrac{9-9}{\sqrt{9} - 3} = \dfrac{9-9}{3-3} = \dfrac{0}{0}$ I.F.

Rationalize the denominator

$$\lim_{x \to 9} \frac{x-9}{\sqrt{x} - 3} = \lim_{x \to 9} \frac{(x-9) \cdot (\sqrt{x} + 3)}{\underset{(A-B)}{(\sqrt{x} - 3)}\underset{(A+B)}{(\sqrt{x} + 3)}}$$

$$= \lim_{x \to 9} \frac{(x-9)(\sqrt{x} + 3)}{\underset{A^2 - B^2}{(\sqrt{x})^2 - 3^2}}$$

$$= \lim_{x \to 9} \frac{\cancel{(x-9)}(\sqrt{x} + 3)}{\cancel{x-9}}$$

$$= \lim_{x \to 9} (\sqrt{x} + 3) = \sqrt{9} + 3 = \boxed{6}$$

Feb 13-9:33 AM

$f(x) = x^3$

Find the difference quotient, evaluate final ans for $h=0$.

$\frac{f(x+h) - f(x)}{h}$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - x^3}{h}$$

$$= \frac{\cancel{x^3} + 3x^2h + 3xh^2 + h^3 - \cancel{x^3}}{h}$$

$$= \frac{\cancel{h}(3x^2 + 3xh + h^2)}{\cancel{h}}$$

$$= 3x^2 + 3xh + h^2$$

For $h=0 \Rightarrow \boxed{3x^2}$

Feb 13-9:39 AM

Evaluate $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ for $f(x) = mx+b$

Linear function

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{m(x+h)+b - (mx+b)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{mx} + mh + \cancel{b} - \cancel{mx} - \cancel{b}}{h}$$

$$= \lim_{h \to 0} \frac{m\cancel{h}}{\cancel{h}} = \lim_{h \to 0} m = \boxed{m}$$

Feb 13-9:46 AM

Try same thing for $f(x) = ax^2 + bx + c$

Make sure you do this before next class.

Feb 13-9:50 AM